A Deterministic Inventory Management Model For Deteriorating Items with Exponential Demand Rate

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ABSTRACT

A deterministic inventory model is developed by assuming exponential demand rate and that items deteriorate with parameter θ . An expression for the average net profit π over one production run has been derived and its optimization with respect to the decision variables Q (initial stock) and T (duration of a production cycle) has been carried out .

Introduction

An extensive research work has already been done by many researchers in the field of deterioration by assuming a constant rate of deterioration and also constant demand. Aggarwal (15) studied an inventory model by considering demand as a function of selling price and three parameter Weibull rate of deterioration. The present paper develops an algorithm for determining the ordering policy when items are deteriorating at a constant rate and demand is increasing exponentially with time. While describing optimum policies for deteriorating items, Ghare and Schrader [5] developed an Economic Order Quantity (EOQ) model by negative exponential distribution by assuming that the instantaneous deterioration rate is constant. A related investigation developed another class of models while assuming that the deterioration rate is time- independent . Mandal and Phaujjdar [12] ,Goswami and Chaudhuri[6],Bose et al. [3], and Mak [11] assume either instantaneous or finite production with different assumptions on the patterns of deterioration. Although these investigations formulated the model for a general time – dependent rate of deterioration, it was actually solved for the simple cases of constant time-proportional deterioration rate. The standard EOQ model assumes a constant and known demand rate over an infinite planning horizon. However, most items experience a stable demand only during their saturation phase of life cycle and for a finite period of time. Furthermore, if the demand rate is deterministic and varies with time, modification of the EOQ model is clearly required. Many studies have expended the EOQ model to accommodate time varying demand patterns. Goswami and Chaudhuri[6][7], Bhunia and Maiti [1], Urban [14] and Bose et al,[3] assumed a linear trend in demand. In a related work, Hong etal.[10] considered an inventory model with time-proportional demand, instantaneous replenishment and no shortage. Dave[4] followed up this effort with variable instantaneous demand ,discrete opportunities for replenishment and shortages. Other investigators ,including Mandal and Phaujjdar [12] , Gupta and Vrat[8] and Su et al.[13], assumed an inventory

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model for stock dependent consumption rate with infinite replenishment rate and without permitting shortages. It can determine the ordering quantity under the circumstances of stock dependent consumption. Hollier and Mak [9] developed inventory replenishment production lot size inventory model with arbitrary production, and demand rate depends on the time function. Goswami and Chaudhuri[7] developed order –level inventory models for deteriorating items in which the finite production rate is proportional to the time dependent demand rate . Furthermore, Bhunia and Maiti [1] assumed that the production rate is a variable and presented inventory models in which the production rate depends on either the on-hand inventory demand or shortages are not allowed at all. A deterministic inventory model is developed by assuming the exponential demand rate and the items deterioration at a constant rate θ. The expression for the average net profit π over one production run is derived and its optimization with respect to the decision variables (initial stock) and (duration of a production of cycle) is carried out . The optimal solution so derived is compared with deteriorating Items with a declining demand . Balkhi and Benkherouf [2] considered deterioration [] case. Sensitivity of optimum solution to changes in parameter values is examined. Several aspects of functional form for the demand rate are considered in a separate section. Finally, the salient features of the problem and its solutions are discussed in brief.

Formulation of the model

We develop here a continuous, deterministic inventory model in which the demand rate at any instant depends on the instantaneous inventory level. The basic assumptions and notations for the model are as follows:

(1) Replenishments are instantaneous.

(2) Lead time (the time between placing an order for replenishment of stock and its receipt) is assumed to be zero. This is a parameter depending on the product as well as the source from which it is available.

(3) The selling price 'S' and the cost 'C' per unit item are known and constant. This assumption implies that no price discounts are offered and no effects of inflation are taken into account.

(4) The procurement cost (or setup cost) C_0 and the holding cost C_h per unit over unit time are known and constant. C_0 is the sum of the cost elements arising from the series of acts starting with the initiation of procurement action and ending with the receipt of the replacement stock. C_0 depends on the product as well as its source of availability. Inventory holding cost C_h is incurred as a function of the number of units of the product on hand and the time duration involved. It is a product-dependent parameter.

(5) No back-orders are allowed.

(6) Demand rate is deterministic and is a known function of the instantaneous inventory level. The functional dependence of the demand rate on the on-hand inventory level *i(t)* is given by

$$R(i)=\alpha e^{\beta t}, \alpha>0, 0<\beta<1 \quad (1)$$

Here α and β are scale and shape parameters respectively. Some comments regarding the choice of the functional form of the demand rate *R(i)* are given.

(7) A constant fraction $\theta(0<\theta\leq 1)$ of the on-hand inventory deteriorates per unit of time.

(8) The time horizon of the inventory system (i.e. the total period over which the inventory process is being studied) is infinite, and T is the cycle length or scheduling period (i.e. the duration of one production run or cycle). The time horizon

being infinitely many production runs, each of duration T.

(9) The inventory system consists of only one product.

(10) There is only one stocking point in a cycle. In each production cycle, replenishment of stock is made only once just at the beginning of the cycle.

(11) π is the average net profit in a production cycle of duration .

Let $Q(>0)$ be the order quantity which enters in to inventory at time t=0.This means $i(t)$ = Q when t= 0. In the interval (0,T), the inventory level gradually decreases mainly to meet demands and partly because of deterioration. By this process, the inventory level at time T falls to i_T (0≠0).The cycle then repeats itself. A pictorial representation of the inventory system is given in fig.1.

Our problem is to determine the optimum values of and which maximize (the net profit). It is to be noted that =0 in the case of deterministic, constant demand rate and instantaneous replenishment [3]. However, the inventory-level-dependent demand rate implies lost sales as the inventory level decreases. Hence, the inventory level should never be allowed to reach zero. We must, therefore, ensure that at time of reordering.

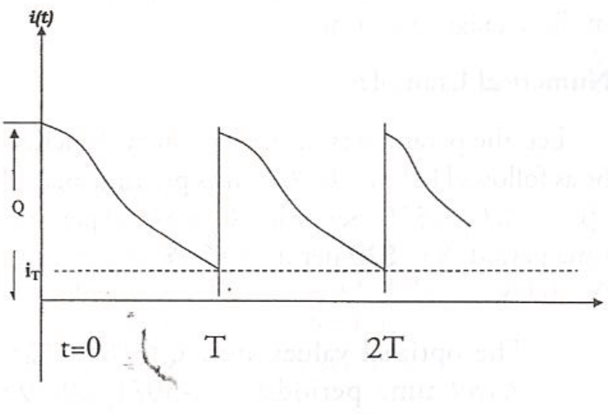

Fig 1. The inventory system

Even if the inventory level drops to zero for any reason whatsoever, the next replenishment should occur immediately upon reaching zero inventory.

The general problem and its solution

The differential equation describing the instantaneous states $i(t)$of in the interval $(0,T)$ is given by

$$\frac{di(t)}{dt} + \theta i(t) = -\alpha e^{\beta t} \quad , \; 0 \le t \le T \qquad (2)$$

which may be written as , $\frac{di}{dt} + \theta i = -\alpha e^{\beta T}$

Integrating we get, $i(t) = -\frac{\alpha}{\theta+\beta} e^{\beta t} + Ke^{-\theta t}$ (3)

where K is constant of integration.

Using the condition, $i(0)$ = Q, we have

$$K = Q + \frac{\alpha}{\theta+\beta}$$

Substituting the value of K , we get

$$i(t) = \frac{\alpha}{\theta+\beta}\left[e^{-\theta t} - e^{\beta t}\right] + Qe^{-\theta t} \qquad (4)$$

The average inventory level is

$$\bar{i} = \frac{1}{T}\int_0^T i(t)\,dt$$

$$= \frac{1}{T}\int_0^T \left[\frac{\alpha}{\beta+\theta} e^{-\theta t} - \frac{\alpha}{\beta+\theta} e^{\beta t} + Qe^{-\theta t}\right] dt$$

$$= \frac{1}{T}\left[-\frac{\alpha}{\theta+\beta}\left(\frac{e^{-\theta T}}{\theta} + \frac{e^{\beta T}}{\beta}\right) + \frac{\alpha}{\theta+\beta}\left(\frac{1}{\theta} + \frac{1}{\beta}\right) + \frac{Q\left(1-e^{-\theta T}\right)}{\theta}\right]$$

The total number of deteriorated items in a cycle is given by

$$D = Q - i_T - \int_0^T \alpha e^{\beta t}\,dt$$

$$= Q - i_T - \alpha \int_0^T e^{\beta t}\,dt$$

$$= Q - i_T - \frac{\alpha}{\beta} e^{\beta T} + \frac{\alpha}{\beta}$$

Noting that $i_T = \dfrac{\alpha}{\beta+\theta}\left(e^{-\theta T} - e^{\beta T}\right) + Qe^{-\theta T}$

this result may be simplified to the form

$$D = Q - e^{-\theta T}\left(Q + \frac{\alpha}{\theta+\beta}\right) + e^{\beta T}\left(\frac{\alpha}{\theta+\beta} - \frac{\alpha}{\beta}\right) + \frac{\alpha}{\beta}$$

The inventory level $i(t)$ at time $t(0 \le t \le T)$ is given by ,

$$i(t) = \frac{\alpha}{\theta+\beta}[e^{-\theta t} - e^{\beta t}] + Qe^{-\theta t} \quad (5)$$

For this result agrees with that of Baker and Urban [14] .

The profit function p is given by

$$\pi = \frac{(S-C)(Q - i_T - D)}{T} - \frac{C_0}{T} - C_h\bar{i} - \frac{CD}{T}, \quad (6)$$

Where $\bar{i}$ is the average inventory level and D is the total number of deteriorated items in (0, T).

The average inventory level is

$$\bar{i} = \frac{1}{T}\left[-\frac{\alpha}{\theta+\beta}\left(\frac{e^{-\theta T}}{\theta} + \frac{e^{\beta T}}{\beta}\right) + \frac{\alpha}{\theta+\beta}\left(\frac{1}{\theta} + \frac{1}{\beta}\right) + \frac{Q}{\theta}\left(1 - e^{-\theta T}\right)\right] \quad (7)$$

The total number D of deteriorated items in (0, T) is given by

$$D = Q - e^{-\theta T}\left(Q + \frac{\alpha}{\theta+\beta}\right) + e^{\beta T}\left(\frac{\alpha}{\theta+\beta} - \frac{\alpha}{\beta}\right) + \frac{\alpha}{\beta} \quad (8)$$

Substituting these values of $\bar{i}$ and D in equation (3), we get

$$\pi = \frac{(S-C)(\frac{\alpha}{\beta}e^{\beta T} - \frac{\alpha}{\beta})}{T} - \frac{C_0}{T} - \frac{C}{T}\left(Q - i_T - \frac{\alpha}{\beta}e^{\beta T} + \frac{\alpha}{\beta}\right)$$

$$- \frac{C_h}{T}\left[-\frac{\alpha}{\theta+\beta}\left(\frac{e^{-\theta T}}{\theta} + \frac{e^{\beta T}}{\beta}\right) + \frac{\alpha}{\theta+\beta}\left(\frac{1}{\theta} + \frac{1}{\beta}\right) + \frac{Q}{\theta}\left(1 - e^{-\theta T}\right)\right] \quad (9)$$

The optimum values Q* of Q and T* of T are obtained by using the necessary conditions

$$\frac{\partial \pi}{\partial Q} = 0 \text{ and } \frac{\partial \pi}{\partial T} = 0 \qquad \text{for a maximum } \pi.$$

These values must satisfy the sufficient conditions:

$$\frac{\partial^2 \pi}{\partial Q^2} < 0 \;,\; \frac{\partial^2 \pi}{\partial T^2} < 0$$

$$\frac{\partial^2 \pi}{\partial Q^2} \cdot \frac{\partial^2 \pi}{\partial T^2} - \left(\frac{\partial \pi}{\partial Q \partial T}\right)^2 > 0.$$

The first condition is

$$\frac{\partial \pi}{\partial Q} = \frac{1}{T}\left[-C\left(1 - \frac{\partial i_T}{\partial Q}\right) - \frac{C_h}{\theta}\left(1 - e^{-\theta T}\right)\right] = 0 \quad (10)$$

$$\text{where } \frac{\partial i_T}{\partial Q} = e^{-\theta T}$$

The second condition is

$$\frac{\partial \pi}{\partial T} = \alpha S\left[\frac{e^{\beta T}(\beta T - 1)}{T^2} + \frac{1}{T^2}\right] + \frac{C_0}{T^2} + \frac{CQ}{T^2} + C\left[\frac{(\partial i_T / \partial T)}{T} - \frac{i_T}{T^2}\right] + \frac{C_h\alpha}{\theta+\beta}\left(\frac{e^{\beta T} - e^{-\theta T}}{T}\right) \quad (11)$$

$$-\frac{C_h\alpha}{\theta+\beta}\left(\frac{e^{-\theta T}}{\theta T^2} + \frac{e^{\beta T}}{\beta T^2}\right) + \frac{C_h\alpha}{\theta+\beta}\left(\frac{1}{\theta T^2} + \frac{1}{\beta T^2}\right) + \frac{C_h Q}{\theta T^2} - \frac{C_h Q}{\theta}\left(\frac{e^{-\theta T}}{T} + \frac{e^{-\theta T}}{\theta T^2}\right)$$

Where

$$\frac{\partial i_T}{\partial T} = -\frac{\alpha\theta e^{-\theta T}}{\theta+\beta} - \frac{\alpha\beta e^{\beta T}}{\theta+\beta} - \theta Q e^{-\theta T}$$

The two simultaneous equations (10) and (11) are now to be solved for Q and T. The solutions are denoted by Q^* and T^* . Substituting the values of Q^* and T^* in Eq.(5) ,we get the optimum value of i_{T^*} of i_T. Lastly substituting the values of Q^*, T^* and i_{T^*} in equation (9),we get the maximum value of the average profit π .

Numerical Examples

Let the parameters of the inventory function be as follows [14]: α = 0.008 units per time period ; β = 0.4; C_0 = \$10 per order ;C_h = \$0.50 per unit time period; S = \$20 per unit; C =\$10 per unit ; θ = 0.05

The optimal values are : Q^*=185.9323, T^*=47.4189 time periods; i_{T^*}=-3071720.795 U and π^*= \$ 890638.8840 per time periods.

It is checked that these values of T^* and Q^* satisfy the sufficient condition for maximizing π. For θ=0, these values are: Q^*=200.000; T^*= 5.5000 time periods; i_{T^*}=199.8395 U and π^* = $-1.417159 per time period

β but T^* remains insensitive in both the cases .Changes in the selling price S induce considerable changes in each of Q*, i_{T^*}, π*, but T^* remains insensitive .The behaviors are similar in the case of

Sensitivity Analysis

Table:-1

Parameter	Changes	Q^*	T^*	i_{T^*}	π^*
α	0.8	-553.5074	47.4189	-3.71738e+08	8.907101e+07
	0.1	100.0378	47.4189	-3.83967e+07	11133811.142
	0.5	-273.4166	47.4189	-1.919836e+08	5.566935e+07
	0.05	146.7196	47.4189	-1.919835e+07	5566868.4890
	0.001	192.4677	47.4189	-383949.22706	111264.6889
	0.003	190.6005	47.4189	-1151883.8056	333942.3950
	0.008	185.9323	47.4189	-3071720.795	890638.8840
β	0.5	185.9344	47.4189	-2.881447e+08	7.899545e+07
	0.4	185.9323	47.4189	-3071720.795	890638.8840
	0.1	185.9315	47.4189	11.25487	-68.09881
	0.0	185.9994	47.4189	17.225878	-36.242952
	0.01	185.9323	47.4189	-3071720.795	890638.8840
	0.03	185.9319	47.4189	16.959196	-70.835442
C_h	0.7	184.3063	47.4189	-3071720.4038	923012.5662
	0.65	184.7128	47.4189	-3071720.3659	914918.5664
	0.6	185.1193	47.4189	-3071720.3279	906824.5822
	0.55	185.5258	47.4189	-3071720.2900	898730.6135
	0.5	185.9323	47.4189	-3071720.795	890638.8840
	0.45	186.3110	47.4189	-3071720.2166	882542.7328
S	26	183.6918	47.4189	-3071720.4612	1327894.1553
	24	184.4387	47.4189	-3071720.3915	1182141.6569
	22	185.1855	47.4189	-3071720.3217	1036389.1586
	20	185.9323	47.4189	-3071720.795	890638.8840
	18	186.6791	47.4189	-3071720.1822	744884.1620
	16	187.4259	47.4189	-3071720.1125	599131.6637

Discussion

We study the sensitivity of Q*, T*, i_{T^*} and π*changes in the values of different parameters associated with the model. It is seen that at each of Q*, i_{T^*} and π* is highly sensitive to change in the values of the scale parameters α and shape parameter, set up cost C_0 .The changes in Q^*, T^*, i_{T^*}, and π* are much sensitive in case of unit cost C . Sensitivity of Q^*, T^*, i_{T^*}, and π* to change in the deterioration rate θ is very significant. Q^*, i_{T^*}, and π* are more sensitive to change in the carrying cost C_h,but T^* remains constant.

Table:-2

Parameter	Changes	Q^*	T^*	i_{T^*}	π^*
C	13	185.1766	47.4189	-3071720.3226	696290.04591
	12	185.4285	47.4189	-3071720.2990	761072.24108
	11	185.6804	47.4189	-3071720.2755	825854.4459
	10.	185.9323	47.4189	-3071720.795	890638.8840
	9	186.1842	47.4189	-3071720.2285	955418.88442
	8	186.4360	47.4189	-3071720.205	1020201.1181
θ	0.05	185.1766	47.4189	-3071720.795	890638.8840
	0.04	185.4285	57.4015	-1.703331e+08	3.931797e+07
	0.02	185.6804	107.3672	-8.539525e+16	9.743122e+15
	0.01	185.9323	207.3502	-2.04515e+34	1.158933e+33
	0.0	186.1842	5.5000	199.8395	-1.417159
	0.1	186.4360	27.5072	-948.379547	486.362854
C_0	12	185.9291	47.4189	-3071720.2523	890636.61939
	11	185.9307	47.4189	-3071720.2521	890636.63986
	10	185.9323	47.4189	-3071720.795.	890638.8840
	9	185.9339	47.4189	-3071720.2518	890636.68082
	8	185.9354	47.4189	-3071720.2517	890636.70133

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